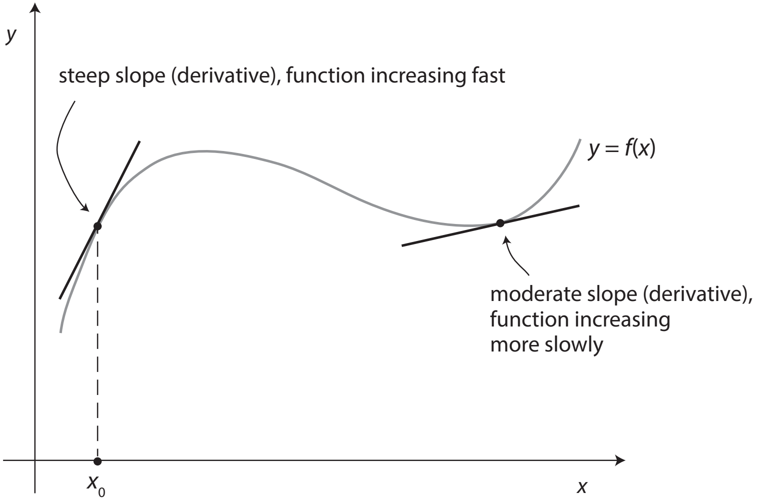

Figure 1.1: The derivative is the slope of the tangent line.

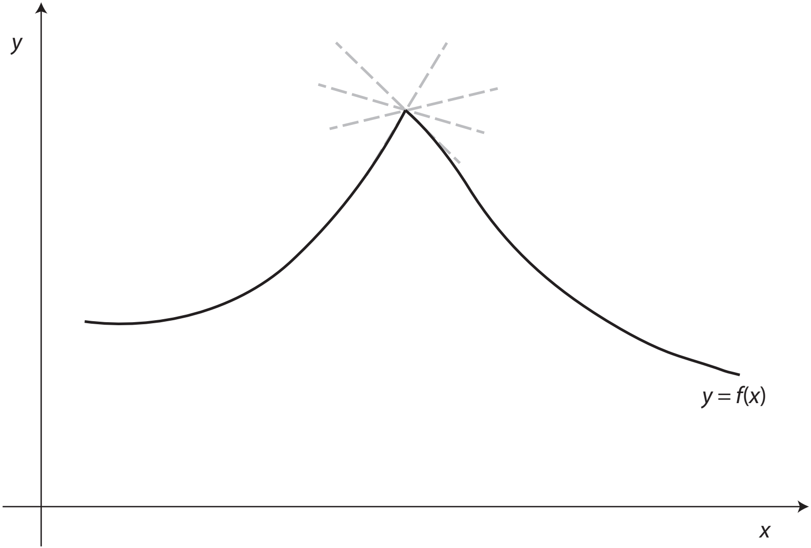

Figure 1.2: A graph with a point where there is no unique tangent line.

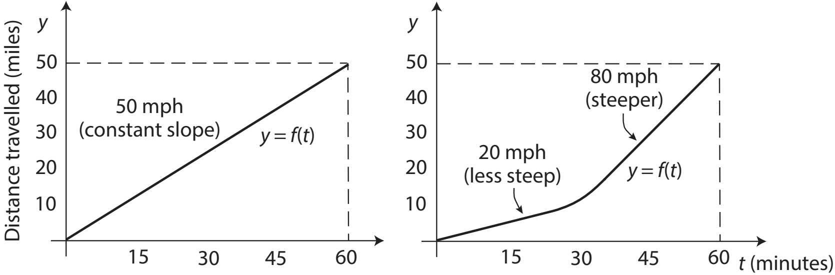

Figure 1.3: Two ways to travel 50 miles in an hour.

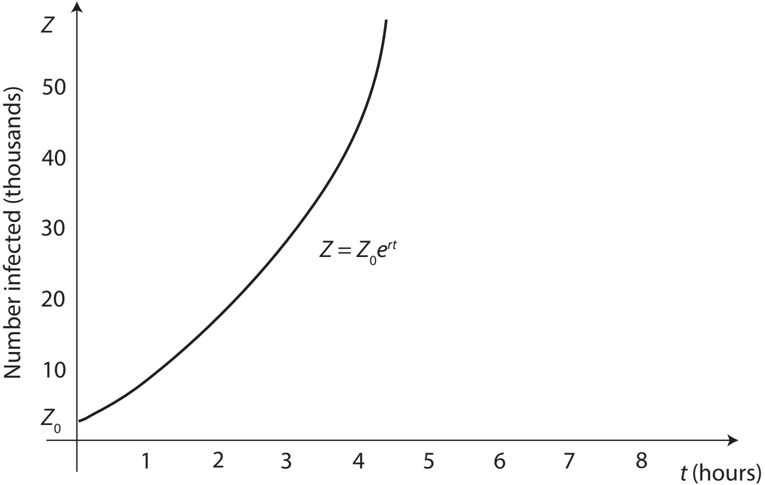

Figure 2.1: Exponential growth.

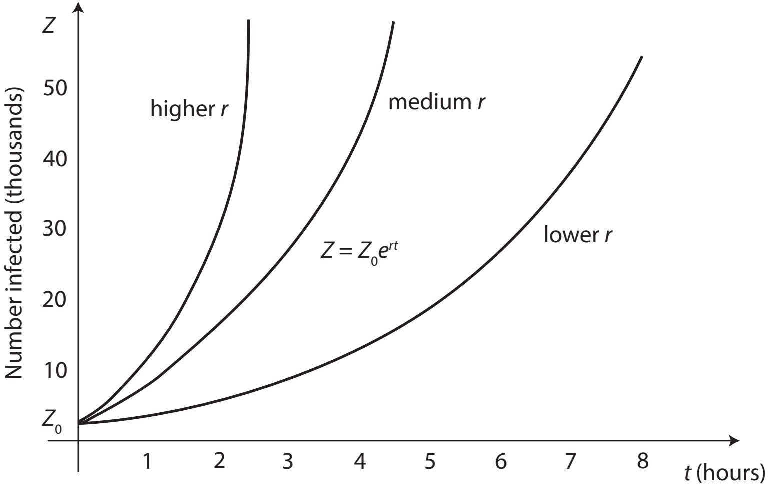

Figure 2.2: Growth rates yield different functions.

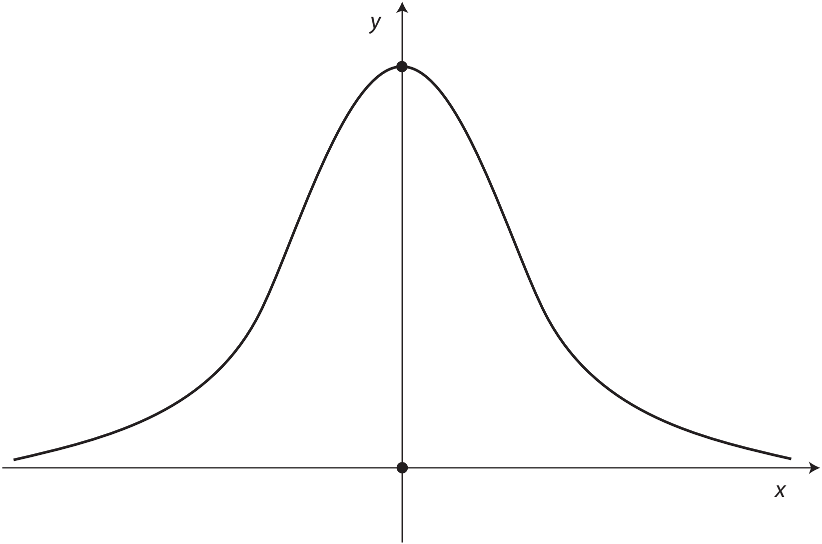

Figure 3.1: The standard normal distribution (bell curve).

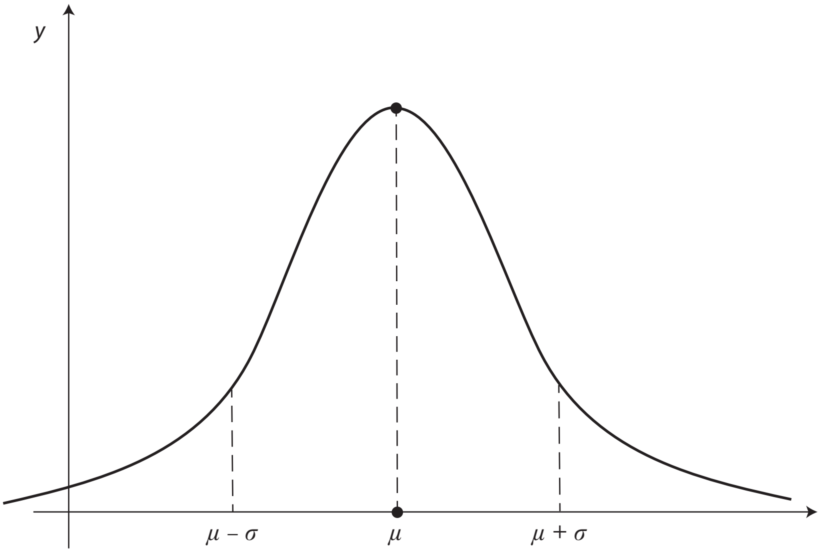

Figure 3.2: The general normal distribution.

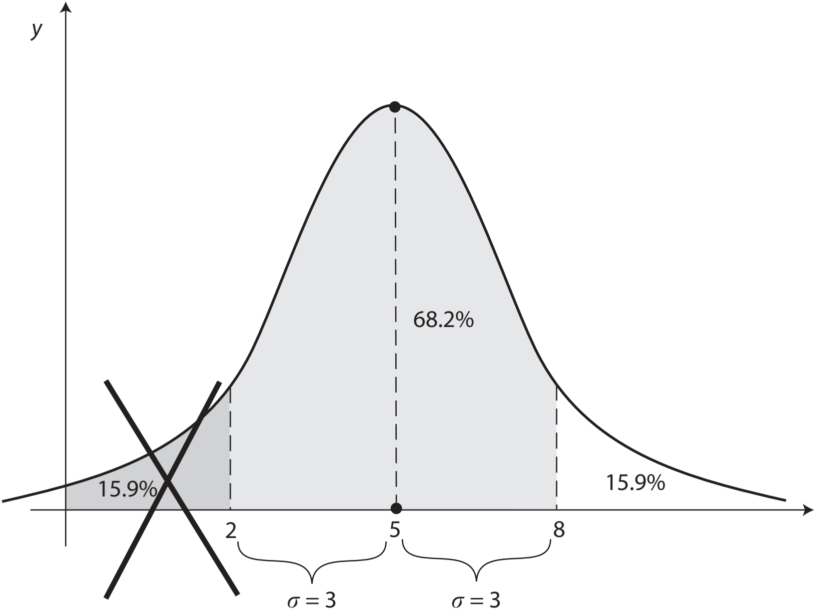

Figure 3.3: The people who run slower than the zombies.

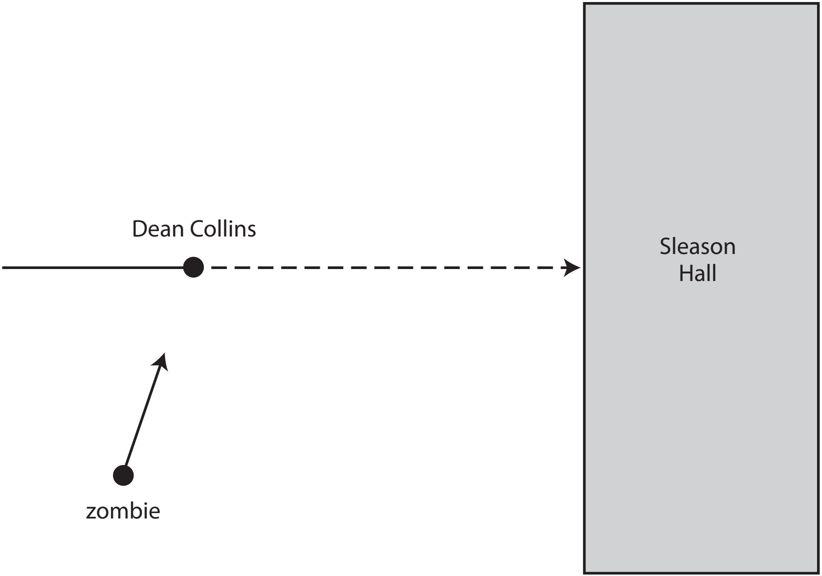

Figure 4.1: The dean's path and the zombie's path.

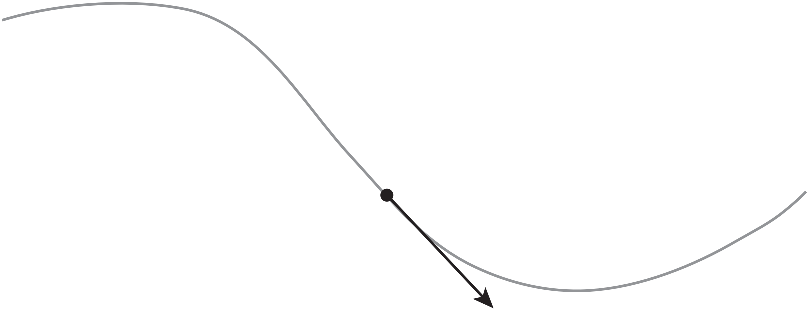

Figure 4.2: The tangent vector to a curve always points in the direction of motion.

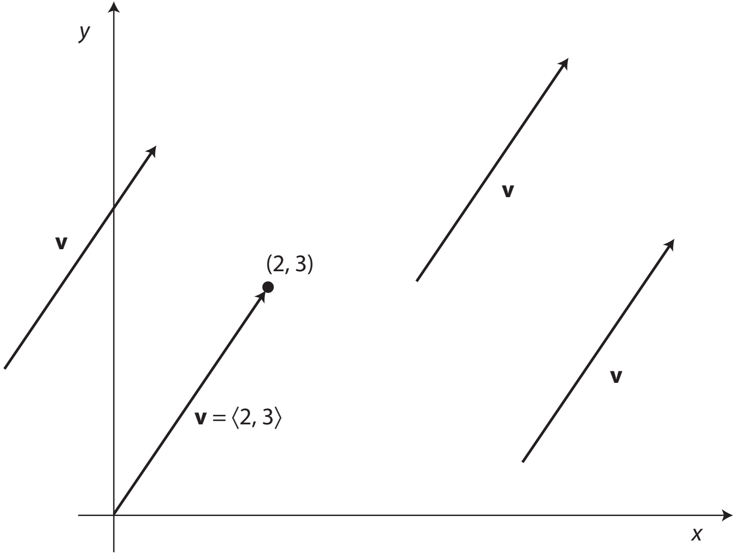

Figure 4.3: A vector has a direction and a length but it can start anywhere.

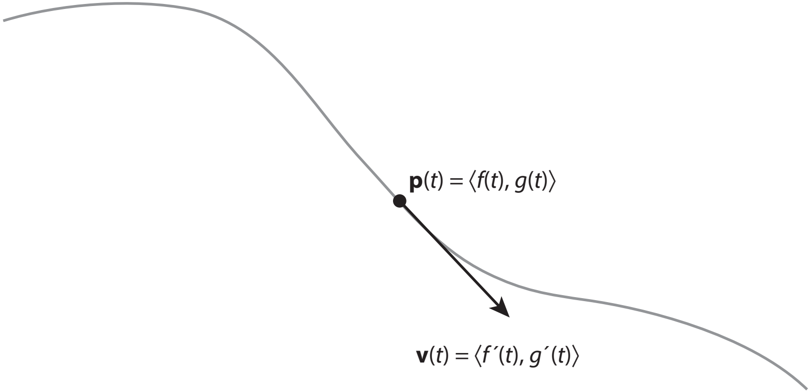

Figure 4.4: The tangent vector is given by $\mathbf{v}(t) = \langle f'(t), g'(t) \rangle$.

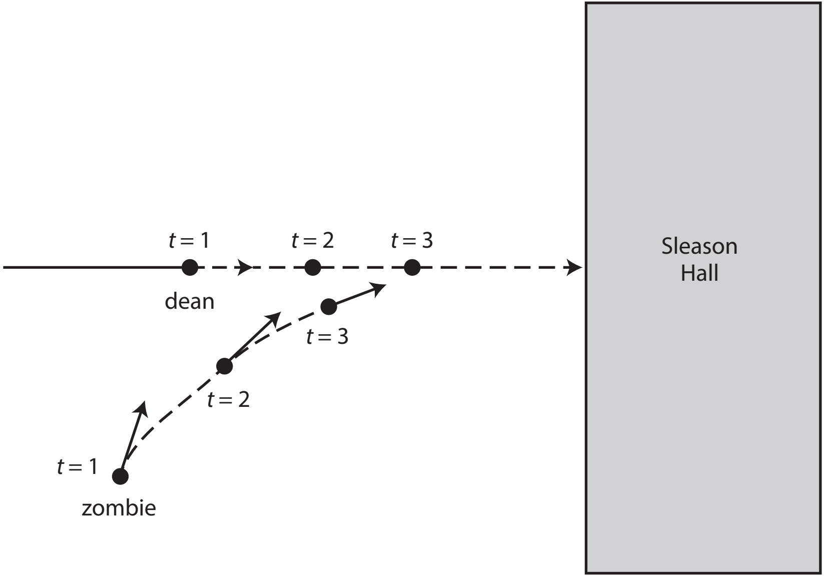

Figure 4.5: The zombie's tangent vector points toward the dean.

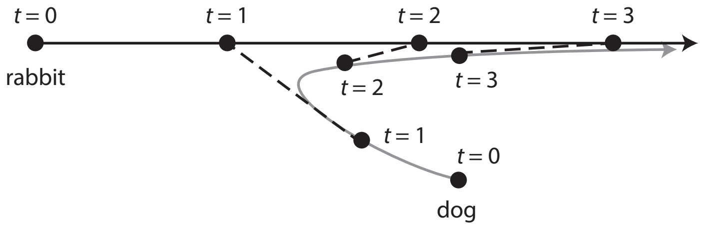

Figure 4.6: Slower dog chasing rabbit that passes nearby.

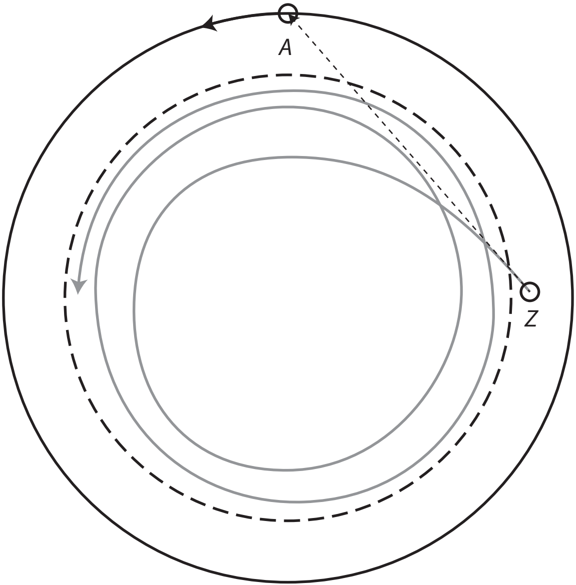

Figure 5.1: Zombie pursuing Angus, who rides his bike in a circle.

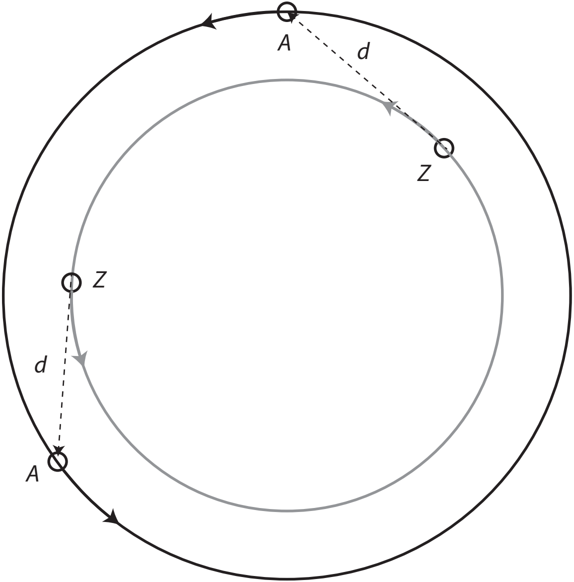

Figure 5.2: On the limit cycle, the zombie's distance to Angus never changes.

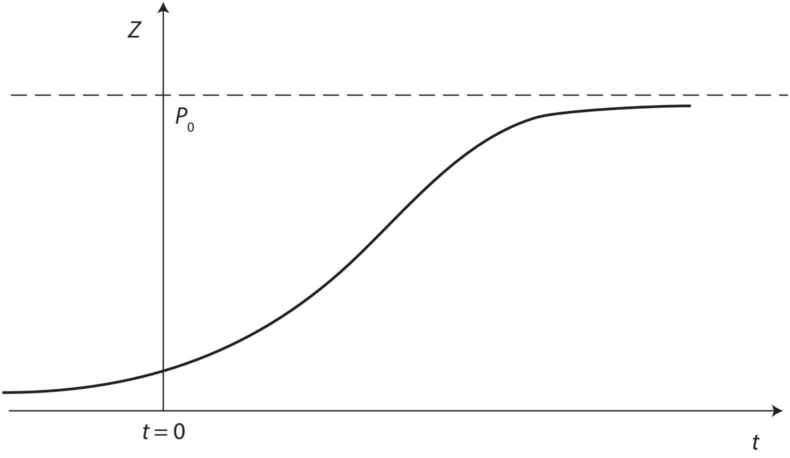

Figure 6.1: Logistic growth of the number of zombies.

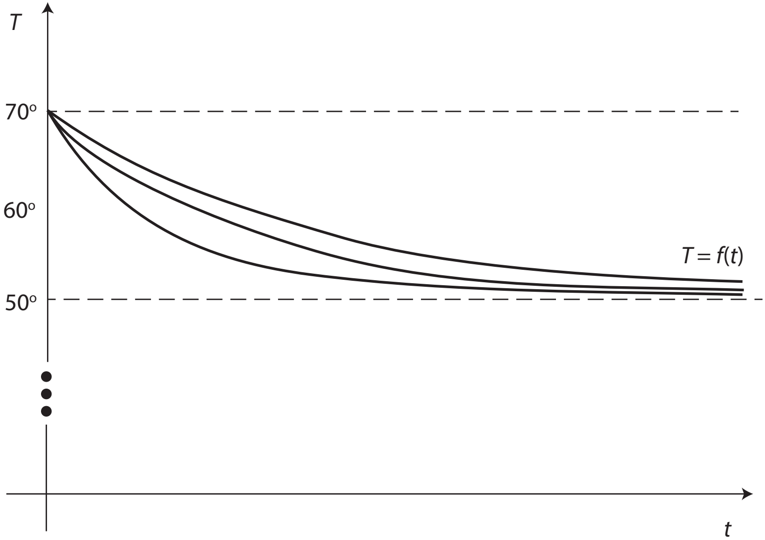

Figure 8.1: Zombie with initial temperature 70° cooling in a room of temperature 50°. Each curve corresponds to a different value of k.

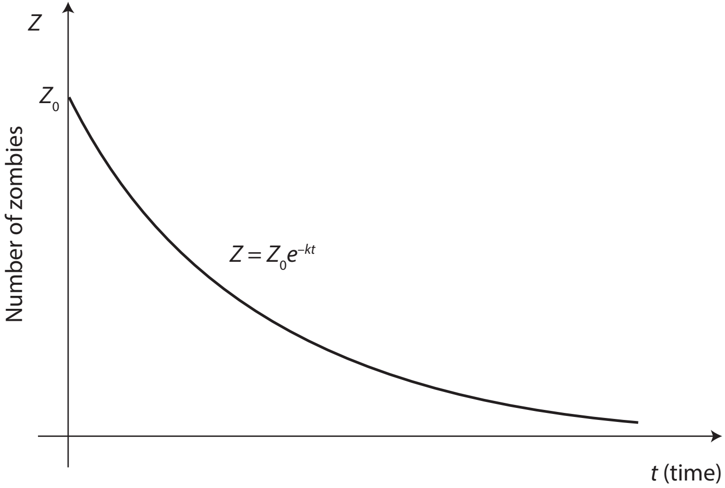

Figure 9.1: The decay in the number of zombies if there are no prey.

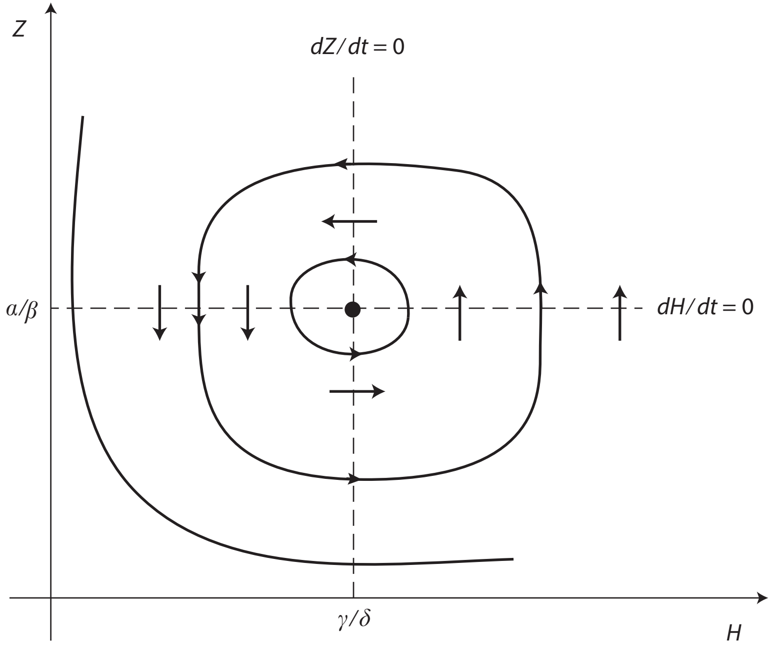

Figure 9.2: Curves in the HZ-plane.

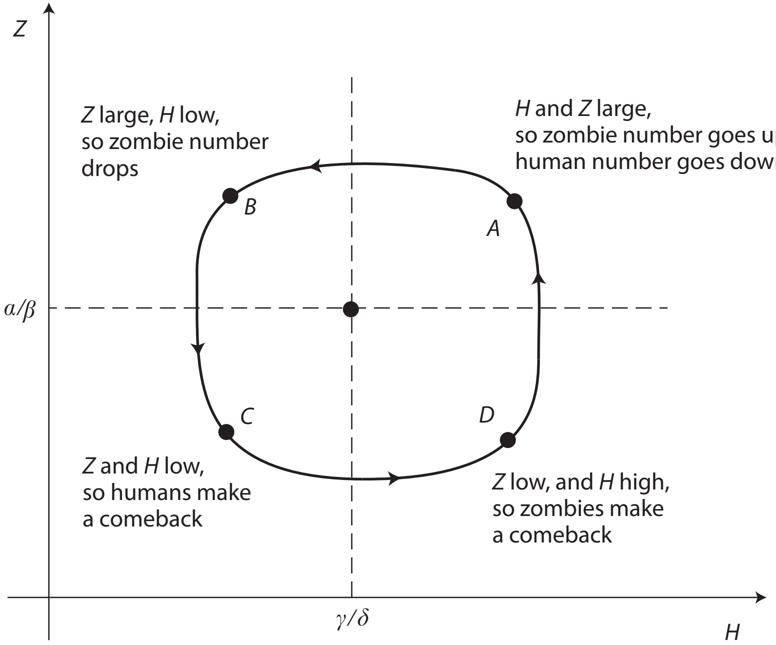

Figure 9.3: The predator-prey curve.

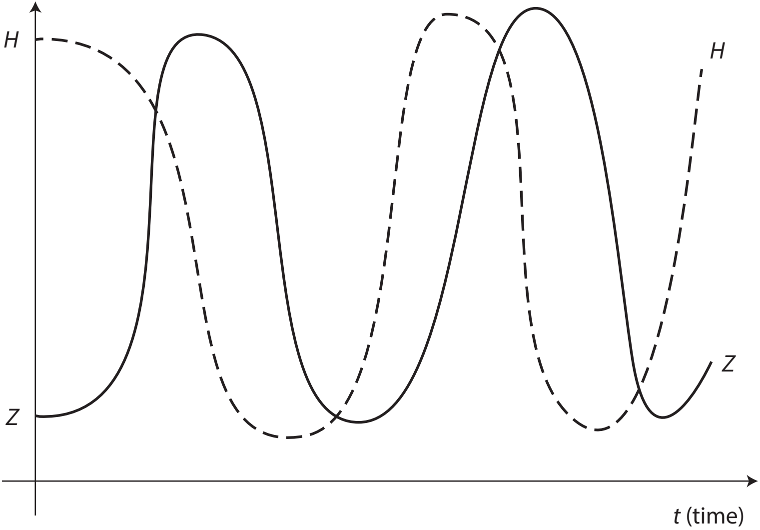

Figure 9.4: The oscillatory behavior of the human-zombie system.

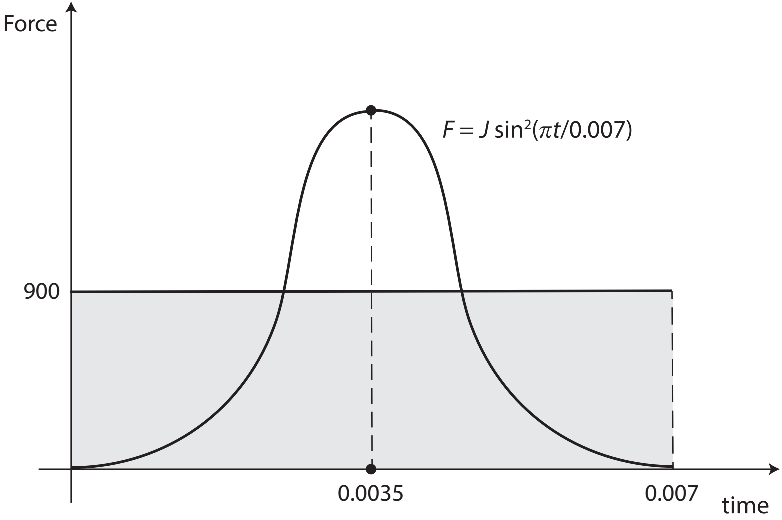

Figure A.1: The force curve.

So

$$\int J \sin^2\left(\frac{\pi t}{0.007}\right) dt = \int J \left(\frac{1 - \cos 2(\frac{\pi t}{0.007})}{2}\right) dt$$

$$= J\left(\frac{1}{2}\int dt - \frac{1}{2}\int \cos\left(\frac{2\pi t}{0.007}\right) dt\right).$$

The first integral is easy, but to do the second integral, we need to use u-substitution. We let $u = \frac{2\pi t}{0.007}$. Then $du = \frac{2\pi}{0.007} dt$, so $dt = \frac{0.007}{2\pi}\, du$ and we get:

$$\int \cos\left(\frac{2\pi t}{0.007}\right) dt = \int \cos u \left(\frac{0.007}{2\pi}\right) du$$

$$= \frac{0.007}{2\pi} \sin u + C$$

$$= \frac{0.007}{2\pi} \sin\left(\frac{2\pi t}{0.007}\right) + C.$$

So we get

$$\int J \sin^2\left(\frac{\pi t}{0.007}\right) dt = J\left(\frac{t}{2} - \frac{0.007}{4\pi}\sin\left(\frac{2\pi t}{0.007}\right)\right) + C.$$

Then, for the definite integral,

$$\int_0^{0.007} J \sin^2\left(\frac{\pi t}{0.007}\right) dt = J\left(\frac{t}{2} - \frac{0.007}{4\pi}\sin\left(\frac{2\pi t}{0.007}\right)\right)\Bigg]_0^{0.007}$$

$$= J\left(\frac{0.007}{2}\right) = J\,(0.0035).$$

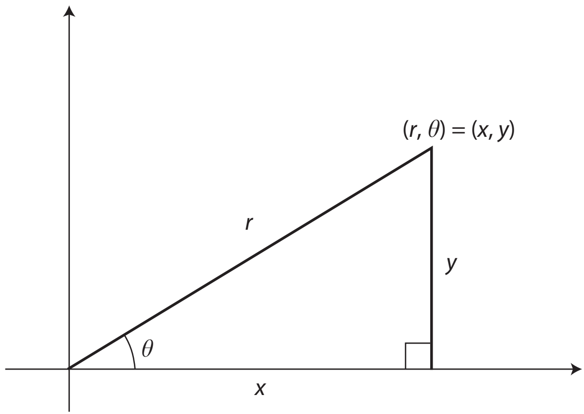

Figure A.2: Polar coordinates.

Then

$$I^2 = \left(\int_{-\infty}^{\infty} e^{\frac{-x^2}{2}}\, dx\right)\left(\int_{-\infty}^{\infty} e^{\frac{-y^2}{2}}\, dy\right)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{-x^2}{2}} e^{\frac{-y^2}{2}}\, dy\, dx$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{-x^2-y^2}{2}}\, dy\, dx.\text{''}$$

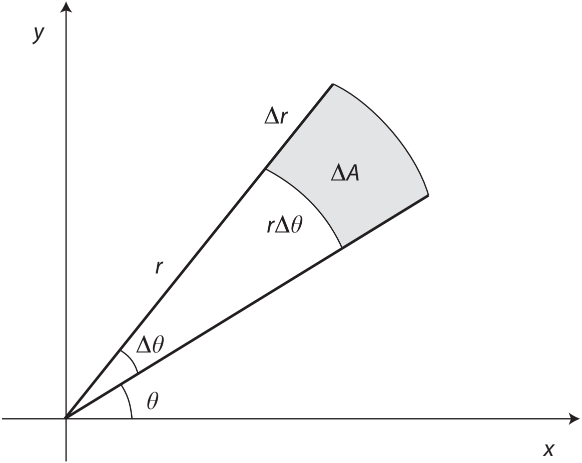

Figure A.3: Varying each coordinate.

"Yeah?"

"Yeah. Then, in the limit, instead of using $dA = dy\,dx$ in the integral, we replace it with $r\,dr\;d\theta$. So we get

$$I^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{-x^2-y^2}{2}}\,dy\,dx$$

$$= \int_0^{2\pi}\int_0^{\infty} e^{\frac{-r^2}{2}}\,r\;dr\;d\theta.$$

Now, we can do the inner r-integral by u-substitution, with $u = \dfrac{-r^2}{2}$. Then $du = -r\;dr$:

$$I^2 = \int_0^{2\pi}\int_0^{\infty} e^{\frac{-r^2}{2}}\,r\;dr\;d\theta$$

$$= \int_0^{2\pi}\int_0^{-\infty} e^u(-1)\,du\,d\theta$$

$$= \int_0^{2\pi} (-e^u)\Big]_0^{-\infty}\,d\theta$$

$$= \int_0^{2\pi} (0-(-1))\,d\theta$$

$$= 2\pi.$$

So $I^2 = 2\pi$ and since I is clearly positive, $I = \sqrt{2\pi}$. That's what we wanted to show."

"That's a pretty cool trick," said Angus.

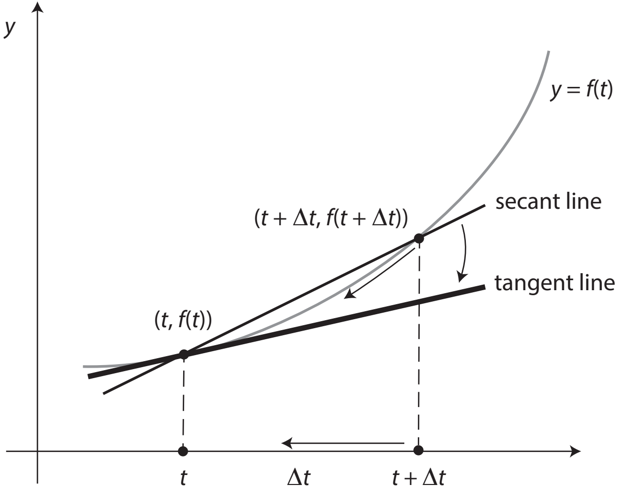

Figure A.4: The definition of the derivative.

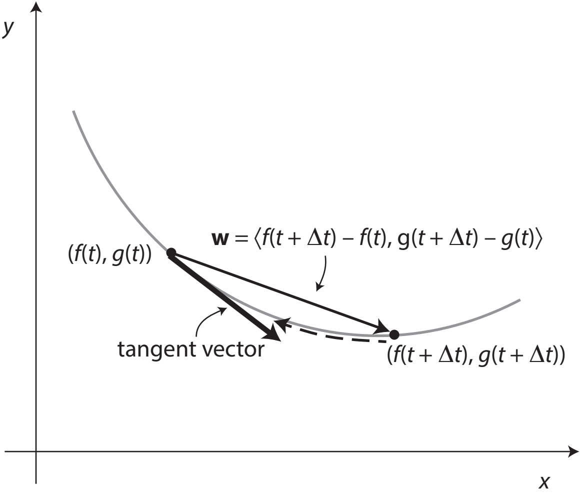

Figure A.5: The velocity vector is tangent.

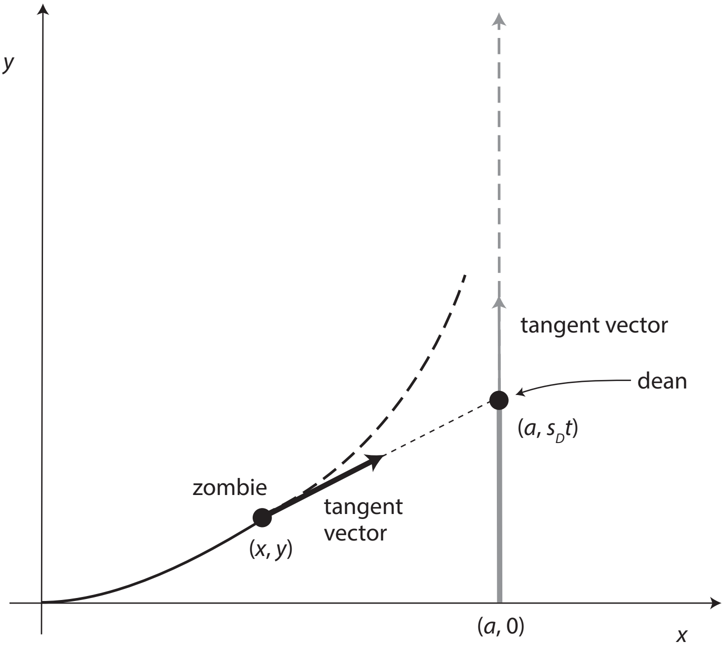

Figure A.6: Zombie chasing the dean.

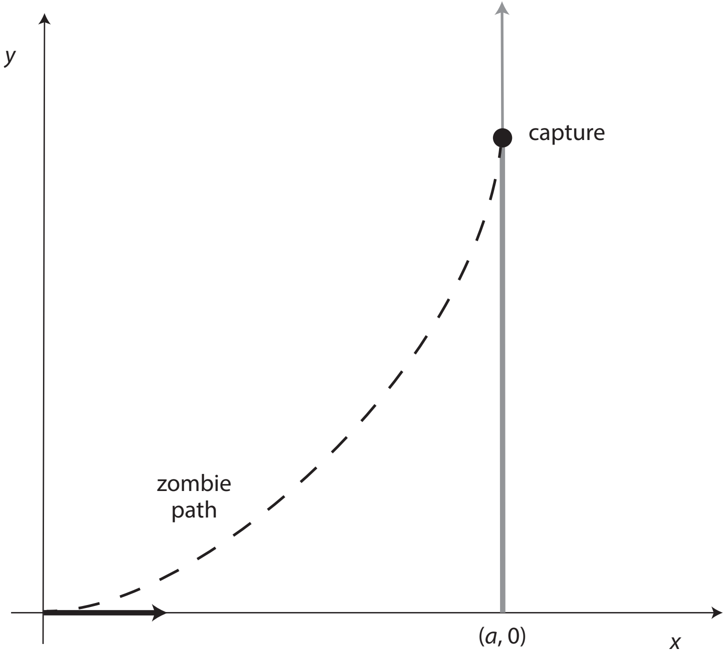

Figure A.7: Zombie catches the dean from directly behind.

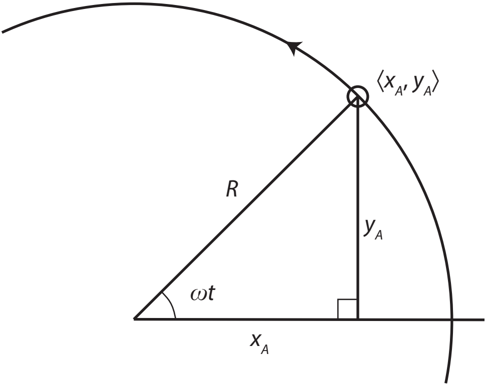

Figure A.8: Angus's path in circular pursuit.

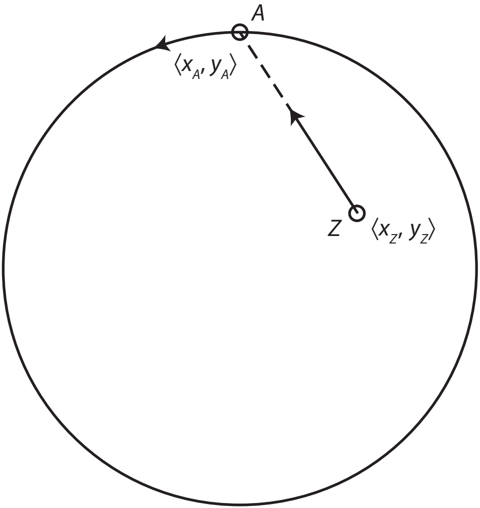

Figure A.9: Determining the zombie's path in circular pursuit.

"By considering each component separately, we obtain two coupled differential equations:

$$\frac{dx_Z}{dt} = s_Z \frac{x_A - x_Z}{\sqrt{(x_A - x_Z)^2 + (y_A - y_Z)^2}},$$

$$\frac{dy_Z}{dt} = s_Z \frac{y_A - y_Z}{\sqrt{(x_A - x_Z)^2 + (y_A - y_Z)^2}}.$$

"Adding in the expressions for x_A and y_A, we obtain:

$$\frac{dx_Z}{dt} = s_Z \frac{R\cos(\omega t) - x_Z}{\sqrt{(R\cos(\omega t) - x_Z)^2 + (R\sin(\omega t) - y_Z)^2}},$$

$$\frac{dy_Z}{dt} = s_Z \frac{R\sin(\omega t) - y_Z}{\sqrt{R\cos(\omega t) - x_Z)^2 + (R\sin(\omega t) - y_Z)^2}}.$$

"So these are the differential equations that need to be satisfied by the zombie's path. But they're too complicated to get an analytic solution. Instead, you get a computer to draw the path."

VII. Logistic growth (continued from p. 84)

"As Jessie explained," I said, "a disease often spreads so that the rate of spread is proportional to the number infected and the number not yet infected. So we have a differential equation that looks like this:

$$\frac{dZ}{dt} = kZ(P_0 - Z)$$

where P_0 is the initial population."

Since Z can be anything, we can set it to whatever we want. So we set it to 0, and we find $1 = AP_0$. So $A = 1/P_0$. Plugging that in, the equation becomes

$$1 = \frac{1}{P_0}(P_0 - Z) + BZ,$$

$$1 = 1 + Z\left(\frac{-1}{P_0} + B\right),$$

$$\frac{-1}{P_0} + B = 0,$$

$$B = \frac{1}{P_0}.\text{”}$$

“And so that means what?”

“That means we now know

$$\frac{1}{Z(P_0 - Z)} = \frac{1}{P_0 Z} + \frac{1}{P_0(P_0 - Z)}.$$

So, now we can integrate:

$$\int \frac{dZ}{Z(P_0 - Z)} = \int \frac{dZ}{P_0 Z} + \int \frac{dZ}{P_0(P_0 - Z)}.$$

“We can do both these integrals. We get

$$\int \frac{dZ}{Z(P_0 - Z)} = \frac{1}{P_0}\ln Z - \frac{1}{P_0}\ln(P_0 - Z) + C$$

$$= \frac{1}{P_0}\ln\frac{Z}{P_0 - Z} + C.$$

"Then $\int \frac{dZ}{Z(P_0-Z)} = \int k\,dt$ becomes

$$\frac{1}{P_0}\ln\frac{Z}{P_0-Z} = kt + C,$$

$$\ln\frac{Z}{P_0-Z} = P_0kt + P_0C,$$

$$\frac{Z}{P_0-Z} = e^{P_0kt+P_0C}.$$

Notice e^{P_0C} is just a constant. Let's call it J. Then we have

$$\frac{Z}{P_0-Z} = J\,e^{P_0kt}.$$

Now solve for Z."

"How do we do that?" asked Angus.

"We multiply through by $P_0 - Z$."

$$Z = (P_0 - Z)J\,e^{P_0kt}$$

$$Z = P_0J\,e^{P_0kt} - ZJ\,e^{P_0kt}$$

$$Z(1 + J\,e^{P_0kt}) = P_0J\,e^{P_0kt}$$

$$Z = \frac{P_0J\,e^{P_0kt}}{1 + J\,e^{P_0kt}}$$

"So that is our function that gives Z as a function of time t."

"In this case, we'll set

$$u = t \qquad \text{and} \qquad dv = e^{-\alpha t}dt.$$

$$\text{Then} \quad du = \frac{du}{dt}dt = dt,$$

$$\text{and} \quad v = \int dv = \int e^{-\alpha t}dt = \frac{-1}{\alpha}e^{-\alpha t}.$$

So according to the formula,

$$\tau = \alpha \int_0^\infty te^{-\alpha t}dt = \alpha \left(t\frac{-1}{\alpha}e^{-\alpha t}\right]_0^\infty - \int_0^\infty \frac{-1}{\alpha}e^{-\alpha t}dt \right)."$$

"But you still have an integral to do," said Jessie.

"Yes, but it's easier than the first one. We can do it directly, and we get

$$\begin{aligned}\tau &= \alpha \left(t\frac{-1}{\alpha}e^{-\alpha t} - \frac{1}{\alpha^2}e^{-\alpha t} \right) \Bigg]_0^\infty \\ &= -\frac{t}{e^{\alpha t}} - \frac{1}{\alpha e^{\alpha t}}\Bigg]_0^\infty \\ &= \lim_{b\to\infty} -\frac{t}{e^{\alpha t}} - \frac{1}{\alpha e^{\alpha t}}\Bigg]_0^b \\ &= \lim_{b\to\infty} -\frac{b}{e^{\alpha b}} - \frac{1}{\alpha e^{\alpha b}} - \left(-\frac{1}{\alpha}\right)."\end{aligned}$$

"Wait," said Jessie. "How do we take the limit of $-\frac{b}{e^{\alpha b}}$? It gives us $\frac{\infty}{\infty}$."

I smiled. "We use L'Hôpital's Rule."

"Why are you smiling?"'

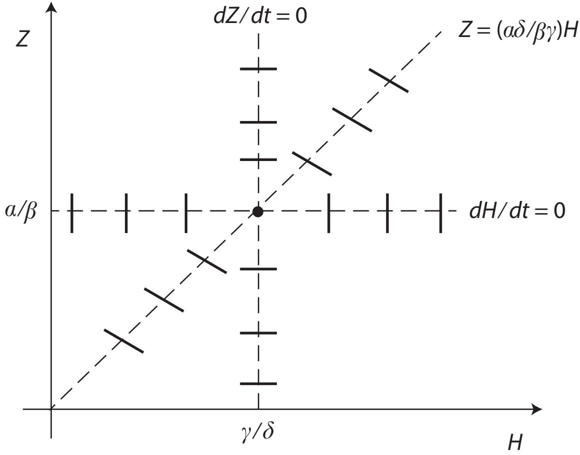

Figure A.10: The slope field in the HZ-plane.

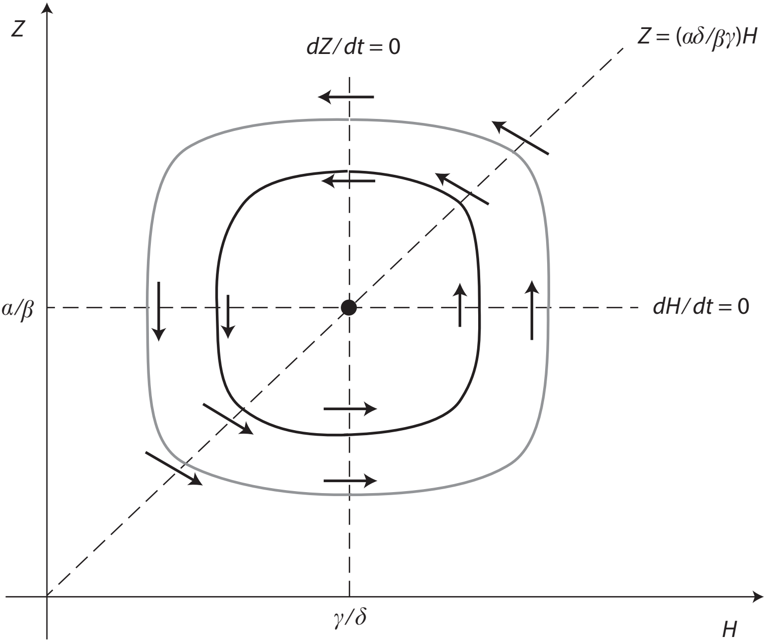

Figure A.11: Curves in the slope field in the HZ-plane.

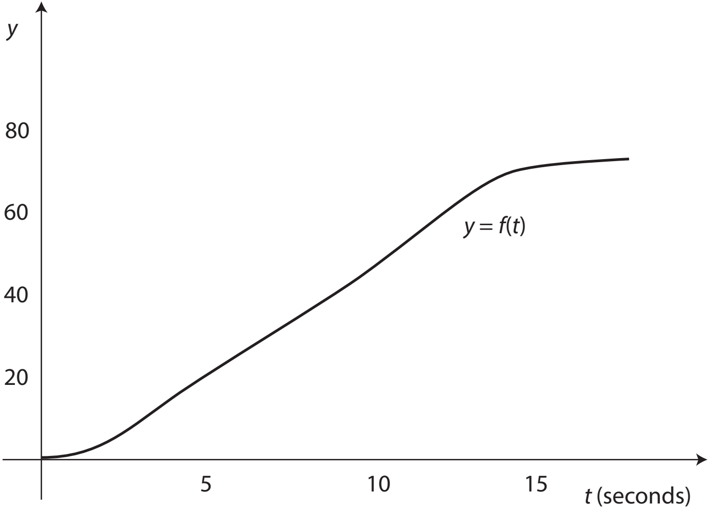

Figure B.1: Distance travelled while running from a zombie as a function of time.

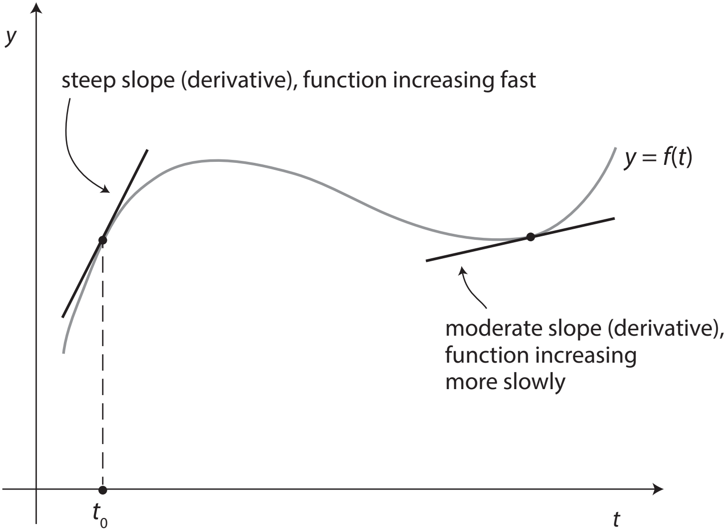

Figure B.2: The slope of the tangent line to a graph measures how steep the graph is at that point.

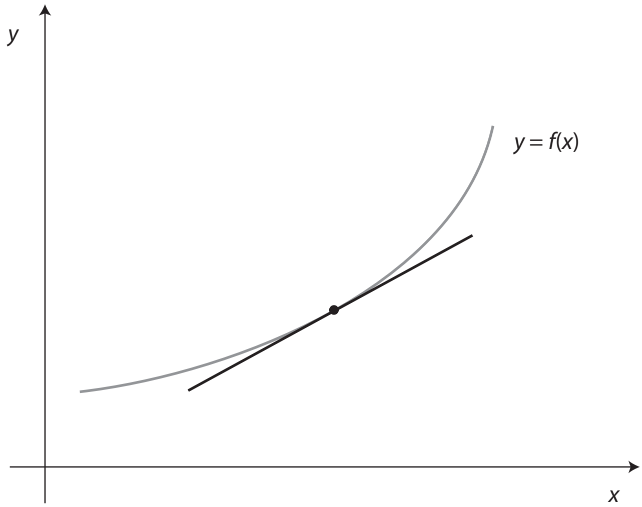

Figure B.3: Positive derivative means positive slope, which means the function is increasing.

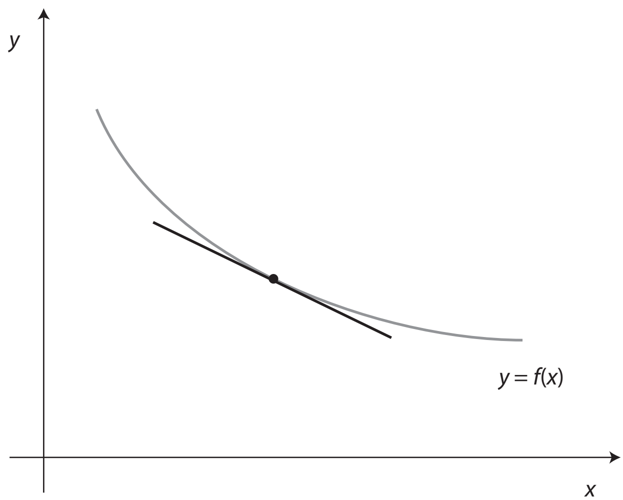

Figure B.4: Negative derivative means negative slope, which means the function is decreasing.

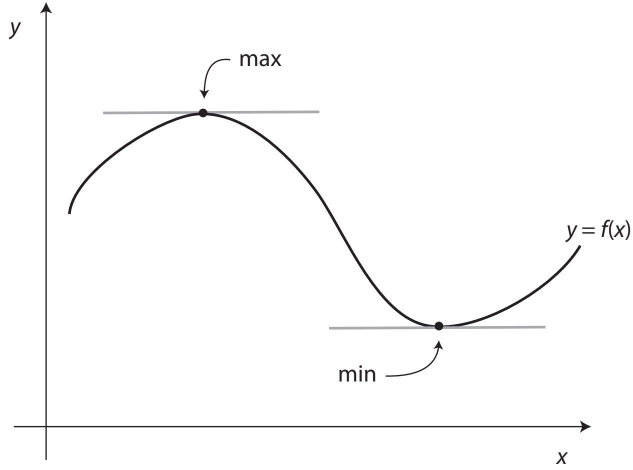

Figure B.5: At a local maximum or minimum, the derivative is 0.

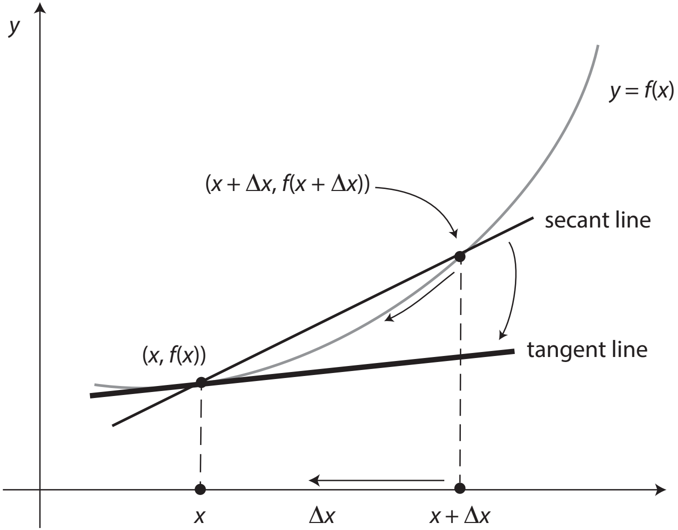

Figure B.6: The slope of the secant line approximates the slope of the tangent line.

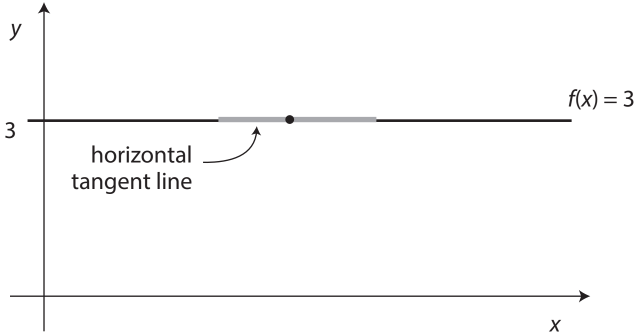

Figure B.7: For a constant function, slope and hence derivative is 0 everywhere.

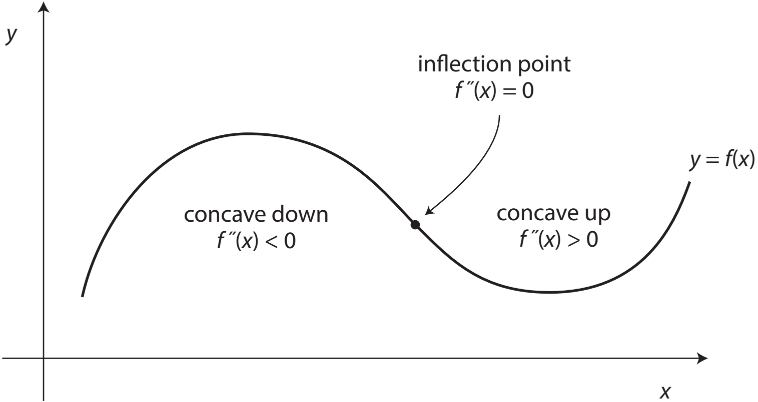

Figure B.8: $f''(x)$ determines concavity.

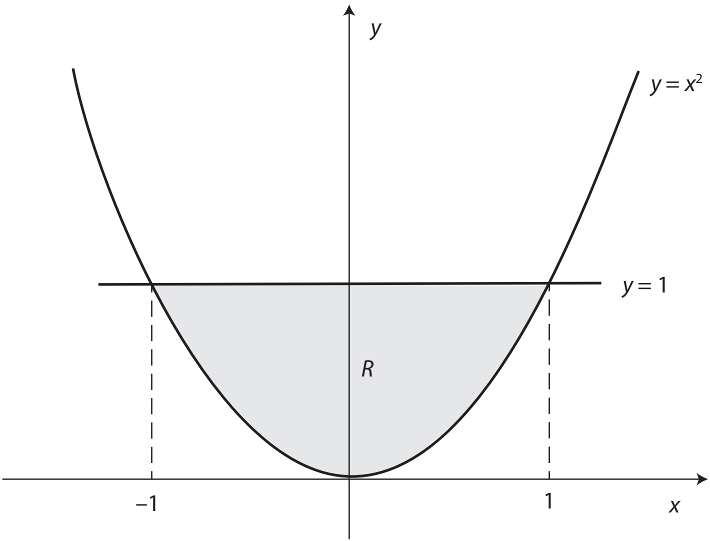

Figure B.9: Integrating over this region *R*.

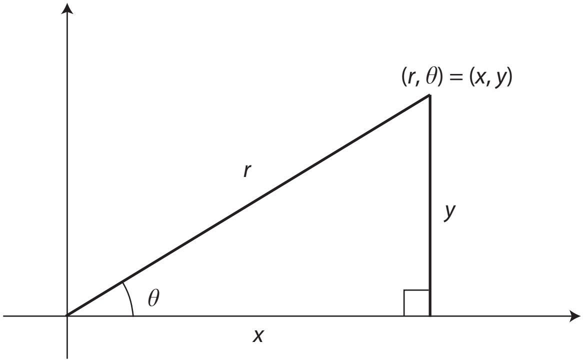

Figure B.10: Polar coordinates.

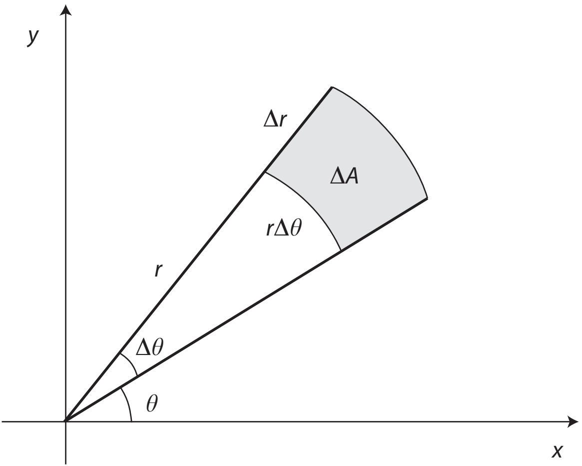

Figure B.11: A small area in polar coordinates.

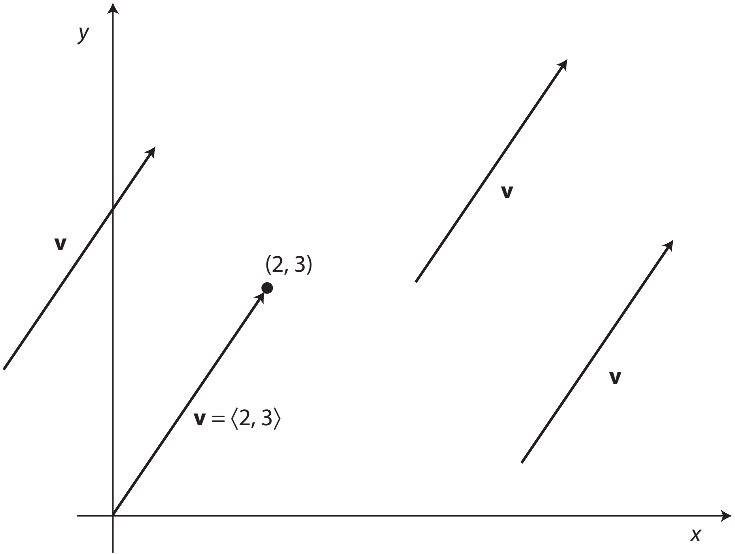

Figure B.12: The vector $\mathbf{v} = \langle 3, 2 \rangle$. It has direction and length.

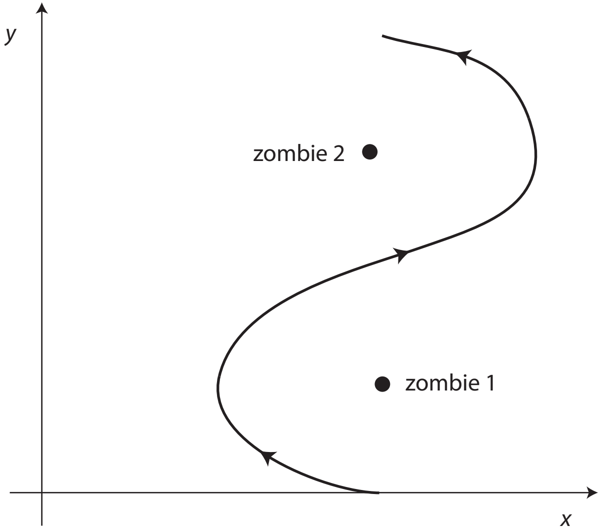

Figure B.13: The path taken by the provost to avoid zombies.

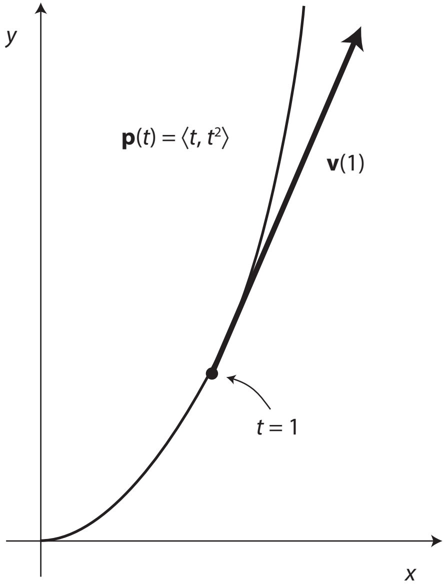

Figure B.14: The provost's parabolic path showing a velocity vector.